Physics GRE Review: Session 1 Notes (partial)

Steuard Jensen

October 5, 2005

1 Optics

1.1 Geometric optics

1.1.1 Explicit equations

For lens and mirror problems, the single most important formula is

$$\frac{1}{o} + \frac{1}{i} = \frac{1}{f} \; .$$

Here, o is the distance from the object to the lens or mirror, i is the distance to the image, and f is the focal length.

The subtle point is keeping track of whether the distances should be positive or negative. For converging lenses and mirrors (convex lenses and concave mirrors), $f > 0$, while for diverging lenses and mirrors (concave lenses and convex mirrors), $f < 0$.

Object distances are positive if they are on the same side of the lens or mirror as the incoming light (this is almost always the case). Image distances are positive if they are on the same side of the lens or mirror as the outgoing light.

Incidentally, the magnification is defined as the ratio of image height to object height. In all cases, it is given by

$$m = \frac{h_{\text{image}}}{h_{\text{object}}} = -\frac{i}{o} \; .$$

The negative sign indicates that when both distances are positive, the image is inverted.

In case the term comes up, a reflected or refracted image is a *real image* if it could be projected onto a screen. A real image has $i > 0$.

1.1.2 Ray tracing

Yes, this section would be more useful if I drew pretty pictures to go with it, but that would be a pain in digital form. So to really understand this, look up the pictures when you do your own review.

"Ray tracing" is often a quick way to see how an optical system works. To solve optics problems this way, just follow the three reference rays:

- Perpendicular to the lens or mirror, then "through" the positive focal point.
- "Through" the positive focal point to the lens or mirror, then perpendicular.
- Straight through the center of the lens without bending at all, or reflect off the center of the mirror as if it were flat.

Perpendicular here means "perpendicular to the center of the lens or mirror".

Positive focal point is a bit more complicated. For mirrors, there's only one focal point. If it's on the same side as the light ray (a converging mirror, with $f > 0$), draw the ray through it. If it's on the opposite side from the light ray (a diverging mirror, with $f < 0$), draw a ray that *would* have passed through it if it were extended through the mirror. (That possibility is why I put "through" in quotes.)

For lenses, the basic idea is the same. If $f > 0$, then the positive focal point is the one on the same side as the light ray; if $f < 0$, it's the one on the opposite side. Just as for mirrors, when $f < 0$ the rays won't ever actually pass through the focal point (but their extensions the other way would).

Mirrors generally have one additional reference ray: • in a direction perpendicular to the mirror, so that the reflected ray comes back right on top of the incoming one. But it's rare that the first three aren't enough.

1.1.3 Compound systems

When you have several lenses in a row (or mirrors, I suppose), the image from one lens can be treated as the object for the next. In this case, you will sometimes find that $o < 0$ because the image falls past the next lens.

In general, ray tracing requires separate reference rays for each lens. But in especially simple systems (e.g. coincident focal points), some reference rays for one lens turn into reference rays for the next. At least some GRE problems can be solved this simple way.

1.2 Diffraction and interference

The single most important formula for these problems is

$$\theta \approx \frac{\lambda}{d} \, .$$

This makes intuitive sense: both phenomena occur when the wavelength of light is no longer small compared to the relevant widths or distances.

This formula is a good limit or approximation for several cases:

- 1-slit diffraction minima: $\sin\theta = m\lambda/d$
- 2-slit interference maxima: $\sin\theta = m\lambda/d$
- Circular diffraction, first minimum: $\sin\theta \approx 1.22\lambda/d$

For 1-slit maxima or 2-slit minima, replace m with $(m + \frac{1}{2})$.

1.2.1 Thin films

Thin film problems involve light passing through some medium (with index of refraction n_0) and hitting a thin film (of thickness d) of some other substance (with $n = n_{\text{film}}$) on a surface of yet another substance (with $n = n_{\text{surface}}$). Some light reflects off of the thin film, and some light passes through it and reflects off of the surface underneath. The problem is generally to find the d that maximizes or minimizes the reflected intensity for a given wavelength λ (or the wavelength for a given d).

There are three main "tricks". First, remember that the light has to go through the thin film both ways: its path length is $2d$. Second, the wavelength depends on n_{film}. If λ is the wavelength in vacuum, the wavelength inside the film is $\lambda_n = \lambda/n$. (We know that the speed of light in the film is $v = c/n$, and $v = \lambda\nu$. As frequency ν can't depend on n, λ must change.)

Finally, watch out for phase changes during reflection! When the initial index of refraction is lower than that of the reflecting surface ($n_1 < n_2$)), the light's phase changes by 180°. (There's no phase change in the opposite case.) For thin films, that effect cancels out if n_{film} is between the other two, but not if it's higher than both of them (or lower than both).

In total, the formula for maximum reflection for intermediate n_{film} is

$$2d = m\lambda_n \, .$$

If you want minimum reflection **or** if n_{film} is higher (or lower) than both its neighbors, use $(m - \frac{1}{2})$ instead of m. For both of these at once, just use m.

1.3 Polarization

Remember that a "plane polarized" wave is one whose amplitude always lies in some single plane. If the direction of the amplitude vector changes with time (beyond just a sign change), that's elliptical polarization; in the limit where it rotates in a circle, that's circular polarization.

Just a few simple rules:

- Going from unpolarized (or circularly polarized) light to plane polarized light cuts intensity by 1/2.
- A polarizer at angle θ relative to plane polarization reduces the wave's *amplitude* according to $A \to A\cos\theta$.
- Intensity $\propto$ amplitude^2.

1.4 Doppler effect

For sound waves reaching a fixed observer with the source moving toward her with velocity u, the observed frequency obeys

$$\nu = \nu_0 \frac{1}{1 - u/v_{\text{sound}}} \ .$$

If the source is fixed and the observer moves toward it, it is instead

$$\nu = \nu_0 (1 + u/v_{\text{sound}}) \ .$$

You can remember which is which by recalling the sonic boom phenomenon: when the source is moving, there can be a singularity when $u = v_{\text{sound}}$.

The relativistic Doppler shift obeys yet another formula, easily derived using 4-vectors. For completeness, that formula is

$$\nu = \nu_0 \frac{1 + u/c}{\sqrt{1 - u^2/c^2}} \ .$$